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DM-003-1164001

Seat No. ________

M. Sc. (Sem. IV) (CBCS) Examination

March – 2022

Mathematics : CMT-4001

(Linear Algebra)

Faculty Code : 003

Subject Code : 1164001

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions : (1) There are total five questions.

(2) All questions are mandatory.

(3) Each question carries equal marks.

1 Answer any **seven** of the following :

(1) Define with example: Singular linear transformation.

(2) Define with example: Companion matrix.

(3) Let $T:\mathbb{R}^3 \to \mathbb{R}^3$ be a linear transformation defined by $T(x_1, x_2, x_3) = (4x_1, 2x_2, 3x_3)$. Justify whether 4 is a characteristic root of T or not?

(4) Define with example: Nilpotent linear transformation.

(5) Define Jordan Canonical Form.

(6) Define with example: Transpose of a matrix.

(7) Define with example: Normal linear transformation.

(8) Define with example: Matrix of a bilinear form.

(9) Define with example: Non-degenerate bilinear form.

(10) Define with example: Unitary Linear Transformation.

2 Answer any **two** of the following :

(1) Let V be a finite dimensional vector space over F and $T \in A_F(V)$. Prove that, T is regular if and only if T maps V onto V.

(2) Let V be an n-dimensional vector space over F. Prove that, $T \in A_F(V)$ is invertible if and only if $m(T)$ is has inverse in F_n.

(3) Let V be a finite dimensional vector space over F and $T \in A_F(V)$. Let $\lambda_1, \lambda_2, \ldots \lambda_k$ are the distinct characteristic roots of T in F and $v_1, v_2, \ldots v_k$ are the characteristic vectors of T beloging to $\lambda_1, \lambda_2, \ldots \lambda_k$ respectively. Prove that, $v_1, v_2, \ldots v_k$ are linearly independent over F.

3 Answer the following :

(1) Let V be a finite dimensional vector space over F and $T \in A_F(V)$. If T is nilpotent, then prove that, $\alpha_0 Id_v + \alpha_1 T + \ldots + \alpha_m T^m$, where the $\alpha_i \in F$, is invertible if $\alpha_0 \neq 0$.

(2) Let V be an n-dimensional vector space over F and $T \in A_F(V)$. Suppose $V = V_1 \oplus V_2$, where V_1 and V_2 are T-invariant subspaces of V. Let T_1 and T_2 be the linear transformations induced by T on V_1 and V_2, respectively. Let $p_1(x)$ and $p_2(x)$ be the minimal polynomials of T_1 and T_2, respectively. Prove that, the minimal polynomial of T is the least common multiple of $p_1(x)$ and $p_2(x)$.

OR

(1) Let the matrix $A = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \in F_3$. Prove that, A is nilpotent and find the invariants of A.

(2) Let V be a finite dimensional vector space over F and $T \in A_F(V)$. Let $p(x) = x^r + \gamma_{r-1} x^{r-1} + \ldots + \gamma_1 x + \gamma_0 \in F[x]$ be the minimal polynomial of T over F. If V is cyclic $F[x]$-module, then prove that, there exists a basis B of V over F such that the matrix of T in B equals $C(p(x))$.

4 Answer the following :

(1) Let $A = \begin{bmatrix} -1 & -1 & -1 \\ 1 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix} \in \mathbb{R}_3$. Determine the Jordan form of A.

(2) State and prove, Cayley-Hamilton Theorem.

5 Answer any two of the following :

(1) Let $A, B \in F_n$. Prove that, $\det(AB) = dt(A)\det(B)$.

(2) Using Cramer's rule find the solutions, in the real field, of the system of equations gioven below :

$x + y + z = 1$

$2x + 3y + 4z = 1$

$x - y - z = 0.$

(3) Let $A \in \mathbb{C}_n$ be a hermitian matrix. Prove that, any characteristic root of A must be real.

(4) Let V be an n-dimensional inner product space over $\mathbb{C}$ and $T \in A_F(V)$. Prove that,

(a) $(S+T)^* = S^* + T^*$

(b) $(\lambda S)^* = \bar{\lambda} S^*$

(c) $(ST)^* = T^* S^*$.